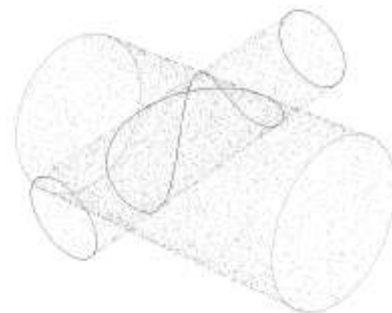
expert, and undefeated champion on the *Wheel of Fortune* game show even though none of the puzzles they gave him were about psychology or superheroes.

Intersecting Cylinders

Michael Lloyd, PhD.
Professor of Mathematics

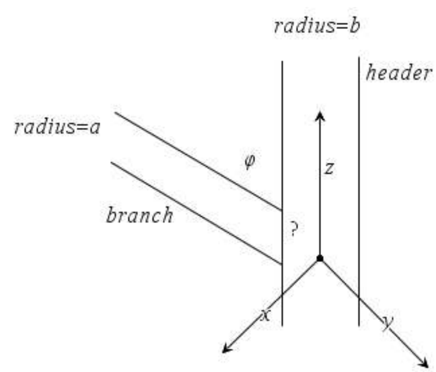
The accompanying picture shows the intersection of two right, circular cylinders. The idea to find a nice algebraic representation for this apparently complicated curve in space was brought to my attention by Fred Worth, whose son was interested in this question from an engineering standpoint.



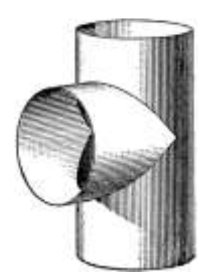
Source: http://virtualmathmuseum.org/SpaceCurves/2cylinders/2cylinders.html

Refer to the accompanying diagram for the symbols used in this paper. Assume without loss of generality that the radii of the cylinders are *a* and *b* and that $0<a\leq b$. Also, assume without loss of generality that the

angle between the cylinders is φ where $0<\varphi\leq\pi/2$. My student, Kyle Walsh, brought the pipe-fitting terminology of *branch* and *header* to my attention. (The header is generally larger than the branch.)



The intersection for $\varphi=\pi/2$ is derived in Gray, A. [*Modern Differential Geometry of Curves and Surfaces with Mathematica, 2nd ed.*] Boca Raton, FL: CRC Press, 1997. To derive the intersection for arbitrary φ, start with equations of cylinders with axes parallel to the *x* and *z* axes, respectively. Note that one of the axes of the cylinders will not intersect if offset *h* is not zero.



Source: http://etc.usf.edu/clipart/62200/62244/62244_planecyl.htm

$$\begin{cases} (y-h)^2+z^2=a^2 \\ x^2+y^2=b^2 \end{cases}$$

Let $R(v,\phi)$be the positive rotation about the positive *y*-axis, and use the linear transformation to get the initial system whose solution is the desired intersection curve.

$$R\left(\begin{bmatrix}x\\y\\z\end{bmatrix},\phi\right)=\begin{bmatrix}\cos\phi & 0 & -\sin\phi\\ 0 & 1 & 0\\ \sin\phi & 0 & \cos\phi\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\begin{cases}(y-h)^2+(x\cos\phi-z\sin\phi)^2=a^2\\ x^2+y^2=b^2\end{cases}$$

Eliminate y^2 and then write in cylindrical coordinates using $r=b$.

$$\begin{aligned}&(\sin^2\phi)z^2\\ &-(2x\cos\phi\sin\phi)z\\ &-2hy+\left(b^2-a^2+h^2-x^2\sin^2\phi\right)=0\end{aligned}$$

$$\begin{aligned}&(\sin^2\phi)z^2\\ &-(2b\cos\theta\cos\phi\sin\phi)z\\ &+b^2\left(1-\cos^2\theta\sin^2\phi\right)-2bh\sin\theta-a^2+h^2=0\end{aligned}$$

To get a solution in cylindrical coordinates, solve for z using the quadratic formula.

$$\begin{cases}r=b\\ z=b\cos\theta\cot\phi\pm\csc\phi\sqrt{a^2-b^2\sin^2\theta+2bh\sin\theta-h^2}\end{cases}$$

Here is the special case when the cylinders have the same radius ($a=b$) and the offset h is zero.

$$\begin{cases}r=b\\ z=b\cos\theta(\cot\phi\pm\csc\phi)\end{cases}$$

Respectively, here is the general solution, and the special case when $a=b$ and $h=0$ in rectangular coordinates.

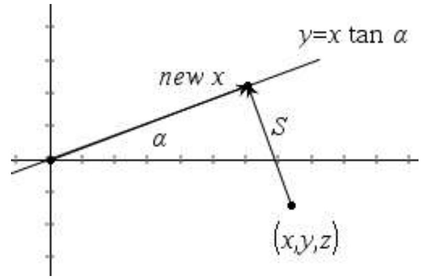
$$\begin{cases}h-a\le y\le h+a\\ x=\pm\sqrt{b^2-y^2}\\ z=x\cot\phi\pm\csc\phi\sqrt{a^2-y^2+2hy-h^2}\end{cases}$$

$$\begin{cases}-b\le x\le b\\ y=\pm\sqrt{b^2-x^2}\\ z=x(\cot\phi\pm\csc\phi)\end{cases}$$

A nice parametric solution can be derived when h=0. Observe that the range of (b/a)sin θ is [-1,1] in the cylindrical solution. Thus, make the substitution $t=\sin^{-1}((b/a)\sin\theta)$, eliminating the plus and minus cases. Only the solution where the branch enters the header is shown since the curve where the branch exits the header can be obtained by reflecting through the origin.

$$\begin{cases}x=\sqrt{b^2-a^2\sin^2 t}\\ y=a\sin t\\ z=\cot t\sqrt{b^2-a^2\sin^2 t}+a\csc\phi\cos t\end{cases},-\pi\le t\le\pi$$

I wanted to make an interactive 3-d graph of the intersection curve on the TI-Nspire. Currently, this software cannot do parametric graphing in space, although this feature is supposed to become available with the release of Version 3.2 in June 2012. Let $S(\mathbf{v},\alpha)$ be the mapping from $\mathbb{R}^3$ into $\mathbb{R}^2$ corresponding to the projection onto the plane $y=x\tan\alpha$. The standard matrix for this linear transformation is shown here.

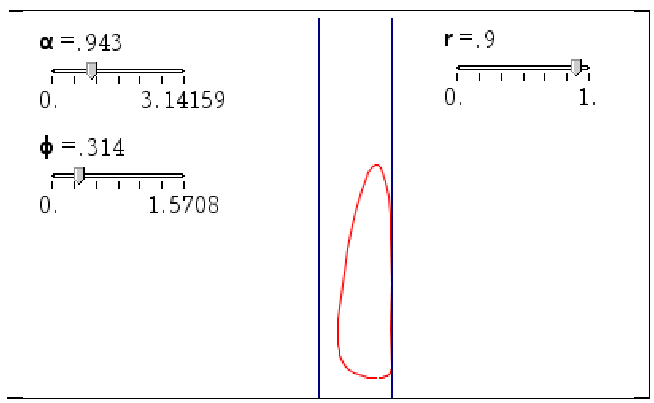


$$S\left(\begin{bmatrix}x\\y\\z\end{bmatrix},\alpha\right)=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}$$

Substitute this into the parametric form of the intersection in space when $h=0$ to obtain the parametric equations for the projection onto a plane. This was entered into the TI-Nspire along with sliders to control the parameters to obtain the interactive graph.

$$\begin{cases}x=\sqrt{b^2-a^2\sin^2 t}\cos\alpha+a\sin t\sin\alpha\\ y=\cot t\sqrt{b^2-a^2\sin^2 t}+a\csc\phi\cos t\end{cases},-\pi\le t\le\pi$$

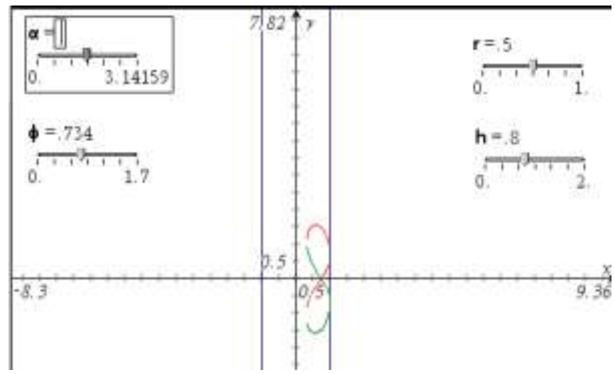
Here is a snapshot of the interactive graph where the graph of the branch is not displayed and the graph of the header is fixed. The parameter α is the viewing angle with respect to the positive x-axis; $r=a/b$, the radius of the branch divided by the header; ϕ is the angle between the branch and the header.



Substituting the projection in the general solution where the offset h may not be zero yields the parametric equations shown here.

$$\begin{cases}x=\cos t\cos\alpha+\sin t\sin\alpha\\ y=\cos t\cot\phi\pm\sqrt{r^2-(\sin t)^2+2h\sin t-h^2}\end{cases},0\le t\le 2\pi$$

This was used to create another interactive application. Unfortunately, this application required a smaller *t* step in order for the graph to appear smooth, and gaps still appear between the plus and minus solutions.



Here are ideas for further research:

- Find a nice parameterization for the general intersection curve(s).
- Redo the above applications using the more powerful 3-d graphing on the Nspire when they become available.

Biographical Sketch

Michael Lloyd received his B.S in Chemical Engineering in 1984 and accepted a position at Henderson State University in 1993 shortly after earning his Ph.D. in Mathematics from Kansas State University. He has presented papers at meetings of the Academy of Economics and Finance, the American Mathematical Society, the Arkansas Conference on Teaching, the Mathematical Association of America, and the Southwest Arkansas Council of Teachers of Mathematics. He has also been an Advanced Placement statistics consultant since 2002.

Hall of Fame or Not?

Fred Worth, Ph.D.
Professor of Mathematics

Abstract - In this paper I will consider three players who do not belong in baseball's Hall of Fame.

I do not recall ever hearing an argument about whether or not someone should be inducted into the football or basketball hall of fame. When those sports have their hall of fame elections, the articles are on the second page of the sports sections and no one really seems to notice. When baseball has its Hall of Fame election, it inspires front page articles in the sports section as well as numerous columns by sports editors and columnists telling why this player should be in or why this player should not be in. People argue about it, get angry about it and rejoice over it. In this essay I am going to talk about three who are in that shouldn't be in.