

Fractal Music

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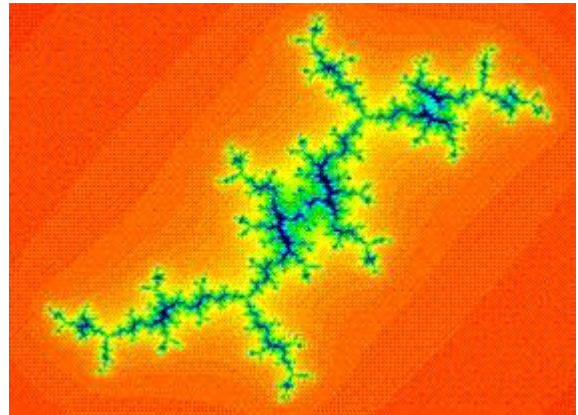
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Abstract

In this paper, the mathematical definition of a couple of fractals will be discussed. Then software will be used to graphically explore various fractals. Lastly, numerical algorithms and fractals can be used to generate music.

Introduction

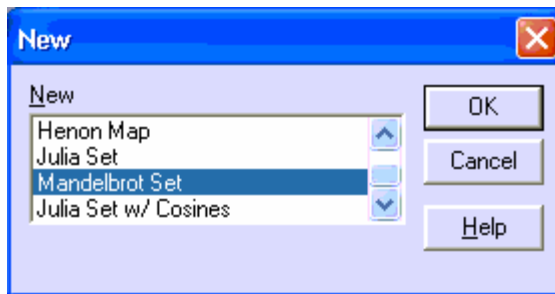
This paper originated as a two-hour workshop that I presented to undergraduates at the Oklahoma-Arkansas meeting of the Mathematical Association of America held at Henderson State University in April 2002. A one-hour version of this talk was also given at the Arkansas Conference on Teaching in Little Rock in November 2002. This graphic displayed here was the background of the opening PowerPoint slide that I used for those two talks. It is a Henon Map fractal created by the software Fantastic Fractal 98 called "electric".



I used the software Oblivion to provide background music before the talk began. A listing of all the software that I used to generate the fractals graphics and music appear at the end of this paper.

The Mandelbrot and Julia Sets

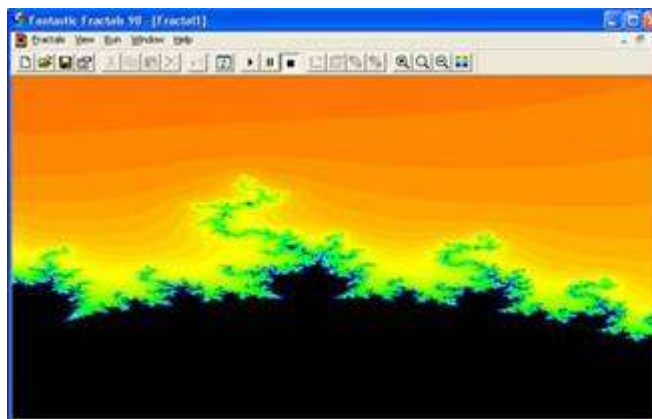
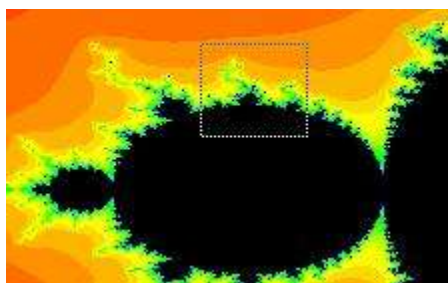
For the first part of the talk, I showed how to explore a fractal using Fractal 98. The MAA workshop participants were all seated in a computer lab that had all the software we would need in a folder called “Fractal” on their desktops. After launching Fantastic Fractals 98, they saw the following dialog box:



The workshop started with the Mandelbrot set because most math students were familiar with it and its definition is relatively easy to understand. Click OK button, and then the Start button to see the entire Mandelbrot set. The black part of the picture is the actual Mandelbrot set.



To zoom in, draw a box holding a left mouse click around and region that you want to explore. This is similar to the zoom box operation on a graphing calculator. The following pictures show the result of zooming once.



The workshop participants spent several minutes exploring by repeatedly zooming in.

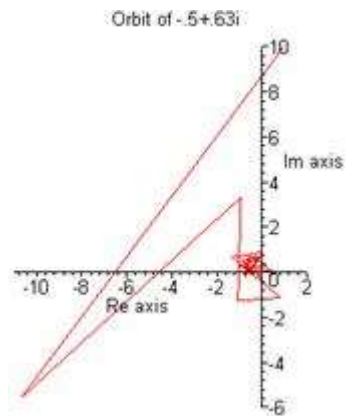
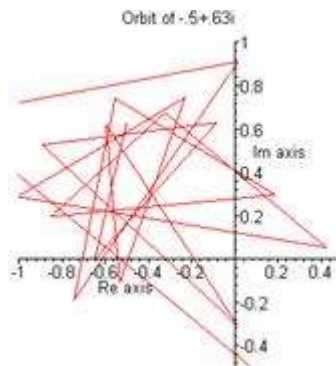
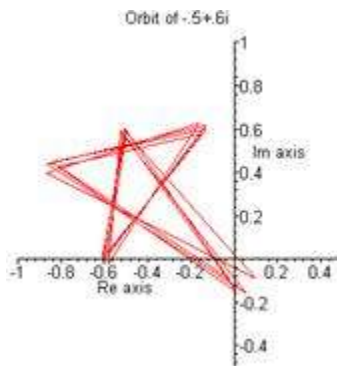
Some of them accidentally made a box with zero width and so the next screen was all one color. In that case, the entire fractal would have to be redrawn. The participants saw surprising shapes as they continued to zoom in. Eventually, however, the boundaries would start smoothing out as the precision of the computer was approached.

Fantastic Fractals 98 has an export to bitmap feature. For most of the software in this paper, it is possible to do a screen capture by pressing Alt-PrtScn.

Definition of the Mandelbrot Set

The Mandelbrot is named after Benoit Mandelbrot (1924 Warsaw – present). It is the set of all

complex numbers c such that the sequence $\begin{cases} z_0 = c \\ z_j = z_{j-1}^2 + c \end{cases}$ is bounded. The first graph shown below is of the sequence (orbit) generated by the complex number $-.5+.6i$. Since the sequence can be confined to a finite disk, this complex number belongs to the Mandelbrot set. It is interesting to that the orbit traces out a fuzzy five-pointed star. The next two graphs are from the orbit of $-.5+.63i$. The sequence appears bounded at first, but the last graph shows the sequence values eventually shoots to complex infinity.



These graphs were generated using a Maple program. The boundedness of the Mandelbrot sequence can also be demonstrated on a graphing calculator that has complex number capability. For example, set the number of decimal places to 1 on the TI-83 and keep hitting ENTER after the second line as shown in the window on the right. The sequence will be bounded no matter how many times the ENTER key is hit.

```

-.5+.6i÷C
Ans²+C
-.5+.6i
-.1+.6i
-.8+.4i
.0-.2i
    
```

If this is done with the other complex number, then it becomes clear the sequence is unbounded after hitting ENTER about 20 times. Note that the color of each pixel in the graph of the Mandelbrot set indicates how fast the sequence diverges.

```

-.5+.63i+C
Ans2+C
      -.5+.6i
      -.6
      -.1+.6i
      -.9+.5i
      .0-.3i
  
```

```

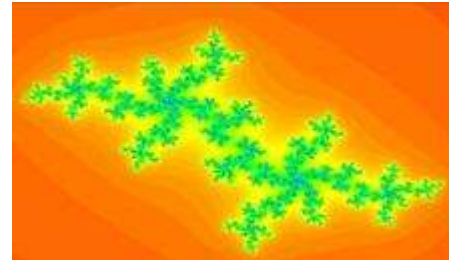
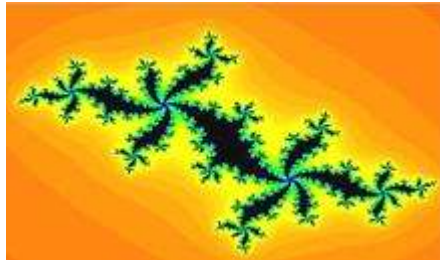
      .0+.9i
      -1.3+.7i
      .8-1.1i
      -1.1-1.3i
      -.9+3.3i
      -10.7-5.5i
      82.4+118.9i
  
```

Definition of a Julia Set

The Julia set was discovered by Gaston Maurice Julia (1893 Algeria -1978 Paris) before Mandelbrot discovered the set that bears his name. For any complex number c , the Julia set J_c is

the set of all complex numbers z such that the sequence $\begin{cases} z_0 = z \\ z_j = z_{j-1}^2 + c \end{cases}$ is bounded. The graphs below are of the Julia sets $J_{-.05+0.6i}$ and $J_{-.05+0.63i}$, respectively.

(These were generated using Fantastic Fractals 98. To enter the parameter c , click on the properties box and enter real and imaginary parts of c .)

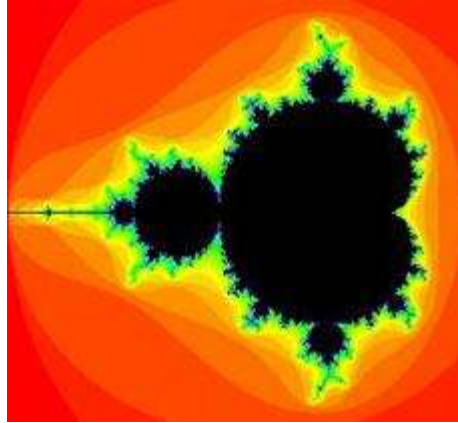


Connection between the Mandelbrot and Julia Sets

The similarity in the definitions of the Mandelbrot and Julia sets suggests that they are intimately related. In fact, the Julia set J_c is connected if and only if c belongs to the Mandelbrot set. Note that the first Julia set displayed above is connected, while the second one is not. An interesting way to graphically demonstrate this connection is to use the Java program called the Julia Movie Maker. Do the following to make a movie:

1. Left click path to choose c 's.
2. Click Make a Movie
3. Click Play or Step

You will see the Julia sets evolve as the points move through the Mandelbrot set and its boundaries.



In May 2002, I gave a workshop for children from a local school ranging from the 2nd to the 6th grade. They had no trouble using the Fantastic Fractals 98 and Julia Movie Maker software.

There are many other fractals you can explore (Iterated Function System, L-System, and Henon Maps) with Fantastic Fractals 98, and other software that you can also use. Fantastic Fractals 98 is the easiest I have found to use and one of the most powerful. Another software, FractInt, is an old DOS program, but it is still rated as one of the best programs for generating images.

Making Music

There is other software not mentioned in this paper for creating fractal music, but I mentioned only those that illustrated my main points, worked well, and were fairly easy to use. This first software gives an idea of how a random sequence of numbers can generate music.

The Music in the Numbers

This software uses the following simple algorithm.

1. Generate a sequence called “count” by counting starting at 0: count = 0, 1, 2, 3, 4 ...
2. Generate the sequence “count \times step + start”.
3. Write the previous sequence in terms of a fixed base.
4. Add the digits of the previous sequence.
5. If you desire, choose a fixed integer larger than 1, and take the modulo of the previous sequence.
6. Choose any scale, and sound the note corresponding to each number in the last sequence.

For example, suppose step=1, start=0, base=3, and modulo=4. Then the sequences described in steps 1 and 2 are the same. The sequences given by the steps 3, 4, 5 appear in the last 3 columns of the following table. (Only a piece of the count sequence between 7 and 8 is shown here.)

count sequence	Convert to base 3.	Add the digits.	Take modulo 4.
7	21	3	3

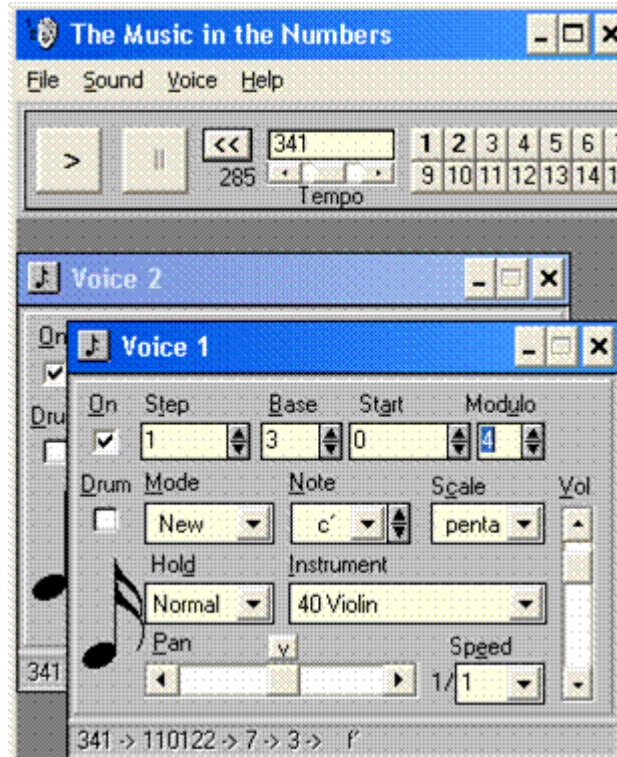
8	22	4	0
9	23	5	1

Most people are familiar with the major scale. It was made famous by the Do-Re-Mi song sung by Julie Andrew's in the musical, The Sound of Music (1965). An mp3 of this song can be found at <http://www.reelclassics.com/Musicals/SoundMusic/soundmusic.htm>.

The pentatonic scale is a nice choice too. A piano teacher told me that she taught it to her learning disabled students, because it sounds nice even if played randomly.

After the parameters for this algorithm are selected in the dialog box, click on the play (>) button. Note that more than one voice can be played at a time.

The status line at the bottom of each voice box shows the numbers used in the algorithm.



This software can also generate a time series graph of the voices. Note repeating pattern has a self-similarity flavor characteristic of a fractal.



The compositions created using this software can be exported to midi and printed in standard musical notation.

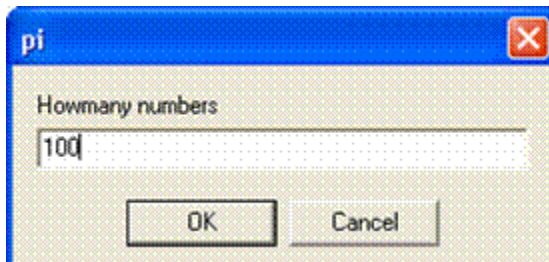


A Musical Generator

This software is more flexible than The Music in the Numbers because it can generate sounds using numerical and fractal data.

Do the following to hear Pi:

1. Click on Cancel/I still want to try it out.
2. Select Data/p/100/OK.
3. Drag the pi icon onto Notes.
4. Click play button.



The jagged graph on the icon is the actual time series graph of the p data.



Here is the Pi song in standard notation:

100 Digits of Pi

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3 1 4 1 5 9 2 6 5 3 5 8 9 7 9 3 2 3 8 4 6 2 6 4 3 3 8 3 2 7
9 5 0 2 8 8 4 1 9 7 1 6 9 3 9 9 3 7 5 1 0 5 8 2 0 9 7 4 9 4
4 5 9 2 3 0 7 8 1 6 4 0 6 2 8 6 2 0 8 9 9 8 6 2 8 0 3 4 8 2
5 3 4 2 1 1 7 0 6 7

I set the Notes minimum = c5 to maximum = e6. Note that this mapped the digits exactly onto the first 10 notes of the C major scale. I wish I had put an 8th rest for the decimal point.

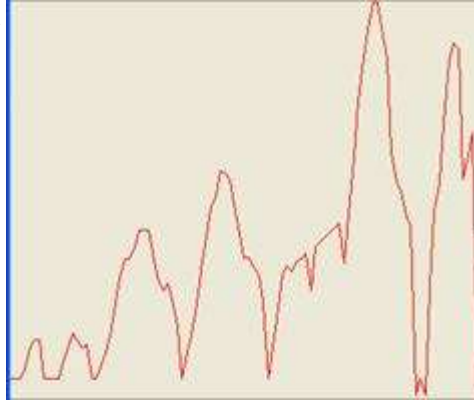
Besides the digits of π , any numerical data set can be converted to music. For example, you can “hear a function”. Define a function $f(x)$ in Maple, and then generate a sequence of function values. Paste the sequence of function values into Notepad to replace all the commas that Maple generates with spaces. Then paste this sequence of numbers into the data dialog box of A Musical Generator.

Another example, I did in the workshop was to use the density of the elements from the periodic table. The densities shown here were exported from the periodic table flash app for the TI-83+. A density of -1 means the density of that element is unknown.

8.80E-05 0.000176 0.534 1.8477 2.34 2.267 0.001165 0.001331 0.001667 0.0008999 0.968
1.738 2.7 2.33 1.823 2.08 0.00298 0.0017824 0.89 1.55 2.985 4.506 6.11 7.15 7.21 7.86 8.9
8.908 8.96 7.14 5.904 5.323 5.727 4.81 3.1023 0.0037493 1.532 2.64 4.472 6.52 8.57 10.28 11
12.45 12.41 12.023 10.49 8.65 7.31 7.265 6.697 6.24 4.63 0.005761 1.8785 3.51 6.162 6.773
6.475 7.01 7.22 7.52 5.244 7.9 8.23 8.54 8.79 9.066 9.32 6.9 9.841 13.31 16.69 19.25 21.02
22.61 22.65 21.09 19.3 13.534 11.85 11.34 9.78 9.196 -1 0.00973 -1 5.5 10.07 11.7 15.37 19.1
20.2 19.816 12 13.51 14.78 -1 8.84

Below is the procedure for using data like this.

1. Select Data/N/type or paste in numbers.
2. Drag onto Notes and proceed as before.
3. Right click on data icon to make a time series plot.

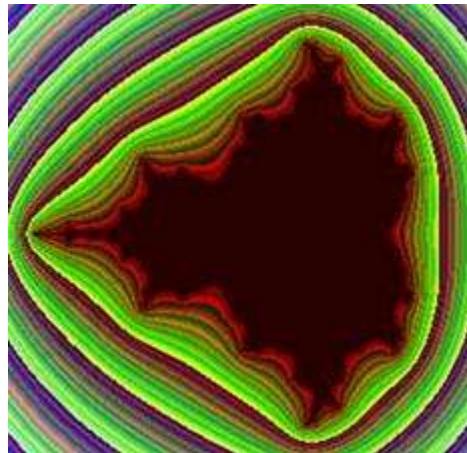


The time series plot for the densities of the elements is shown on the right.

If you want to import a numerical data file into A Musical Generator, then give the file the extension prn and separate the numbers with a space. To open such a file, click on folder icon, and open files of type numbers. Any text files can also be converted to music – the software will use the ASCII values. You could convert a chapter of Shakespeare to a song!

A Musical Generator can also be used to hear 2-dimensional fractals, 1-dimensional fractals, dynamic systems and complex maps. Do the following to hear the Mandelbrot set:

1. Click on Complex Maps/Mandelbrot set.
2. Double click on icon and change the Estimation method to Continuous potential method.
3. Drag onto Notes and proceed as before.



The Well Tempered Fractal

This software is older and has some compatibility problems with Windows, but has a large palate of sounds and scales to work with. space = next
F1-F10 = select fractal type
m = music toggle

The basic commands are:

left, right = select scale
q = quit

The function keys at the top of the keyboard are used to select the fractal that is used to generate the image and music:

The music must be turned off when selecting new fractals. I have not figured out how to hear the exported midi files or capture the screen images.

F1 = Mira fractal
F2 = the KAM Torus
F3 = the Hopalong
F4 = the Seahorse or Hippocampus fractal
F5 = Chebyshev fractal
F6 = Ikeda Attractor
F7 = Julia fractal
F8 = Ivanov fractal
F9 = Stewart-Golubitsky icon fractals
F10 = Duffing Attractor

Fractal Image Software

1. Fantastic Fractals 98 (<http://www.techlar.com/fractals/websys.exe?file=download/software.html>)
2. The Mandelbrot Movie Maker (<http://math.bu.edu/DYSYS/applets/JuliaMovie.html>)
3. FractInt 20.0 (<http://www.fractint.org/>)

Music Software

1. The Music in the Numbers (<http://reglos.de/musinum/>)
2. A Musical Generator (<http://www.musoft-builders.com/links/amg.shtml>)
3. Oblivion (<http://tinpan.fortunecity.com/fluke/807/>)
4. The Well Tempered Fractal (http://www.hitsquad.com/smm/programs/Well_Tempered_Fractal/download.shtml)

Other Software

1. Maple (<http://www.maplesoft.com/flash/index.html>)
2. Noteworthy Composer (<http://NoteWorthyComposer.com>)
3. Microsoft Photo Editor
4. TI Connect (<http://education.ti.com/product/software/ticonnect/features/features.html>)
5. Periodic Table of the Elements for the TI-83+ (<http://education.ti.com/product/tech/83pse/apps/apps.html>)

Biographical Sketch

Michael Lloyd received his B.S in Chemical Engineering in 1984 and his Ph.D. in Mathematics from Kansas State University in Manhattan. He has presented papers at meetings of the

Mathematical Association of America, the Arkansas Conference on Teaching, and the American Mathematical Society. He has been at Henderson State University since August 1993.

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