Workshop on Mathematics in Industry

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Abstract

This is a report of the proceedings of the workshop that I attended in July and the conclusions reached in regard to curriculum and career planning.

The Purpose of the Workshop

The mathematics department at the University of San Diego provided a web page that summarized the purpose of the workshop. For most the workshop, speakers with diverse job experience described their work, and explained some mathematical problems they've worked on. At the end of conference, some of those speakers and additional mathematicians gave a panel discussion with an emphasis on how math educators should modify their curriculum and teaching to prepare math students for employment in industry. There were many opportunities to ask the speakers questions during their presentations, or even talk one-on-one between presentations or at lunch.

In this report, I will first give the conclusions reached almost unanimously by the speakers. Then I will summarize each of the speakers' talks including some of the mathematics they did. At the end of the report, I'll talk about the panel discussion. This entire report may seem like a hodgepodge, but that is how much of the conference was!

What Applied Math Students Need to Know

There was consensus among the speakers and panel that students training for a mathematics job in industry should have the following skills:

- Be self-motivated.
- Have problem solving ability.
- Be able to think logically.
- Be able to work in a team.
- Be able to communicate.
- Possess computer skills.
- Have seen diverse applications in mathematics, probability, statistics, and science.

I will elaborate on these areas in the next section. Exactly **how** to change the curriculum in order to instill these skills in our students was not fully addressed in the conference.

The Six Speakers

Dr. Raymond Roan (GTE Government Systems)

Dr. Roan worked for Hughes aircraft before he started working for GTE, a company that deals with communication and information systems. I spoke with Dr. Roan during one of the breaks and we had a long conversation during one of the workshop lunches. He had been trained in theoretical mathematics, earning his Ph.D. in harmonic analysis in 1981. He thought applied mathematics was uninteresting until he discovered the following problem: Use the quadratic formula to solve $x^2 - 80{,}000x + 1 = 0$. The two roots are approximately 80,000 and 10^{-5}. The accuracy of the smaller root is going to be poor unless your computer has high precision because significant digits will cancel in the numerator of the usual quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

I remember from a numerical analysis class that this problem is remedied by using an alternate form of the quadratic formula:

$$x = \frac{2c}{-b \mp \sqrt{b^2 - 4ac}}$$

This formula can be derived by rationalizing the numerator of the usual quadratic formula. A related problem he found interesting is the sensitivity of the roots of a polynomial to changing its coefficients. Changing one coefficient of a Wilkinson's polynomial with roots 1, 2, 3, … 20 by its least significant bit will make the roots larger than 8 move far off the real axis.

Dr. Roan began his talk by asking the audience what characterized math majors. We came up with the following:

- Detailed oriented.
- Enjoy challenges, math is fun.
- Disciplined.
- Tenacious.
- Has intellectual curiosity.
- Creative.
- Thinks outside the boxes.
- Symbol manipulator.
- Able to memorize. (This may be good or bad.)
- Likes to play games, often mental ones.
- Likes to communicate math.
- Dislikes writing papers.
- Likes freedom. (That is, hates labs.)
- More interested in the process than the answer.
- Is a "nerd", and isolationist, withdrawn from the world.
- Is strange or unusual.

- Is capable of learning a difficult subject.

Dr. Roan added the last two items. I have noticed some of these traits in our math/computer science majors, and also in myself! Dr. Roan said that math majors or minors should gain the following from their education:

- Have breadth. That is, they should know a variety of problem solving techniques.
- Have seen applications. For example, physics.
- Be able to organize and communicate thoughts.
- Have Computer skills.
- Be comfortable with "new" problems.
- Think with abstractions. That is, be able to distill general knowledge from details.

He said that experts (human or an artificial intelligence) have a library of simple examples wired in their brain. He added that math students should be able to do the following after graduation:

1. Learn to think
    a. Abstractly (identify the heart of the matter).
    b. Analytically/logically.
2. Learn to solve problems and enjoy it!
    a. Refine problem solving intuition and skills.
    b. Develop mental discipline (the ability to focus).
3. Learn to recognize when a process is correct and complete.
    a. Be able to teach themselves.
    b. Be versatile.
    c. Be flexible.

Dr. Roan said that although some hiring managers realize this, many need to see that their best people are mathematicians. Math educators need to put out a good product to change the minds of these hiring managers. Dr. Robert Roe, who was on Dr. Fred Worth's Ph.D. committee, said that math for liberal arts courses might help the attitude of the students in those courses who become hiring managers. Here's a list Dr. Roan gave which describes the character necessary for a technical job worker in industry.

1. Be able to communicate
    a. In writing.
    b. Orally to large groups.
2. Be a self-starter. That is,
    a. Self motivated.
    b. Self taught.
3. Be a team player.
4. Be flexible.

Here are the critical issues in industry:

- Schedule and budget. A company does not care "if a solution exists". It is more interested

in an adequate or partial solution. (Students who ask for partial credit would like this!)
- Computer skills. This is the "ticket" that will help a math graduate find employment. In fact, he said it is impossible to survive in industry without computer skills. He recommended fluency in at least one language. He mentioned C++, Java, UNIX, and training in object-oriented design and analysis.

He warned that the above list was colored by his particular job experience.

Here are some of the "real world" assignments that Dr. Roan has worked on:

- Determining the range from a radar to target.
- Triangulation involving speed and environment. Can we assume a flat earth?
- Multi-target and multi-source tracking.
- Correlating the tracks.
- Determining if a point is in a region.
- Algorithm prototyping.

He elaborated on the first and third examples.

Dr. Roan mentioned a fact about Gauss that I will mention when I teach statistics: Around the year 1800, Gauss predicted the location of an asteroid after it had disappeared from the sights of the astronomer's telescopes using a least squares technique. The technique was not officially published until twenty years later by Legendre.

Dr. Laura Bloom (Agouron Pharmaceuticals)

Dr. Bloom is a research scientist for the pharmacology company, Agouron. She was educated as a mathematician, but was hired by Agouron because the hiring manager could not find a biologist who had good analytical skills. (Perhaps Henderson's biology department should take heed in regard to their majors' math requirements.)

She said that hot areas of employment are

- Bioinformatics – finding patterns in chemical/biological information.
- Biostatistics – determining whether a medical treatment is useful.
- Computational chemistry – predicting how a drug will bind to a target.

These areas require knowledge of probability, statistics, and computer programming.

Skills that our math majors should have:

- Problem solving – analytical skills and being able to solve diverse problems.
- Flexibility – being able to think across disciplines.
- Computation – programming, numerical analysis
- Communication – spoken and written communication skills.
- Teamwork – adeptness at working with colleagues and also being able to work

independently.

She noted that writing a clear mathematical proof is similar to writing a clear memo. Also, our graduates should put on their resume that they have done group work in their classes.

Dr. Bloom gave examples of real advertisements she had seen. Many job advertisements in industry do not mention that the company is looking for a mathematician, but she said that a mathematician could do many of these jobs. For example, an advertisement for a position in computational biology at Du Pont said their company was looking for "system's engineers, computer scientists, and computational biologists". It seemed to me that having confidence is important when looking for this kind of employment. Dr. Bloom recommended that a student looking for a job in a specialty area should learn how to "speak the language" and then drop "buzz words" during an interview. Reading a professional journal like the Journal of Theoretical Biology will help the student learn the lingo.

Dr. Bloom and several of the workshop speakers said that getting an internship makes it much easier to get a job in industry. A couple of web sites listing internships are http://www.math.arizona.edu/~restrepo/AMII/internships.html and http://www.amstat.org/education/internships.html. Profiles of industrial mathematicians can be found at http://www.ams.org/careers/. SIAM's Mathematical Sciences Career information is at http://www.siam.org/careers/.

She gave the following examples of problems she has worked on at Agouron:

- Modeling and simulation work for an anticancer project.
- Modeling and simulation of biochemical kinetics (two problems)
- Reproducing the phenomena for one pair of drugs.

Part of Dr. Bloom's job is to do internal consulting, that is, consulting within her company. Here is a list of skills needed and problems she has dealt with within her company:

1) Computation

1. Significant digits versus decimal places.
2. Relative standard deviation.
3. Finding the line that describes the linear portion of the data on a plot.

2) Statistics

1. ANOVA versus t-test
2. "The computer lets me calculate a standard deviation with 3 data points, so why isn't that enough data?"
3. How to use statistical and graphical software.

3) Curve-fitting

1. Meaning of R^2, the coefficient of determination.
2. Curve-fitting and initial parameter values, multiple minima, lack of convergence, etc.
3. Effect of improper rounding on curve and surface fitting.
4. Visualization of 3-D data.

I have been emphasizing significant digits in algebra and trigonometry the last few times I have taught them. Dr. Bloom gave the example of a biologist who recorded the concentrations 1, .5, .3, .06, .03 as he diluted the solution **by half** at each step! The relative standard deviation in the above list refers to comparing the mean and standard deviation. In Statistical Methods, we use the coefficient of variation $CV = \bar{x} / s$ to do this. Dr. Bloom recommended that statistics be taught **in** mathematics departments. From her point of view, curve fitting is very important. I found it interesting that Jim Hall, the senior quality engineer for Scroll technologies told me that qualitative analysis is the most important part of statistics.

At the end of her talk, Dr. Bloom gave several examples which could be used in the classroom from the areas of calculus, algebra, geometry, and statistics.

Mr. Dale Honeycutt (Environmental Systems Research Institute)

Mr. Honeycutt was the only speaker who was not a mathematician. The Environmental Systems Research Institute (ESRI) creates software for companies so that they can make efficient use of network infrastructure like roads, rail lines, telephone lines, water lines, and sewer lines.

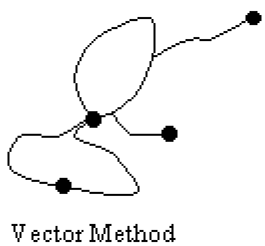
An interesting example of this occurred during the Reagan years: The plan was to have a roving herd of semi trucks roaming over a large area in the desert so that every spot is visited with equal probability. When the Russians sent ballistic missiles over the pole, the herd would disperse and each truck would hunker down waiting for the atomic warheads to detonate. After the attack, each truck would launch its missile toward Russia.

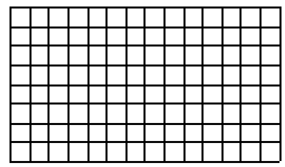
Confirming what Dr. Bloom said, ESRI is a company that never advertises for mathematicians, but hires them. A recent applied math graduate would be encouraged to hear Mr. Honeycutt say, "The current supply of qualified professionals is not enough to meet demand". The people that are hired in Graphical Information Systems (GIS):

- Software engineers (Must be able to program in C++.)
- Product specialists
- Release managers
- Product managers
- Consultants

Teamwork is important, and ironically, Mr. Honeycutt said that he preferred to work with "yellers", that is, people who are loud and vocal. Other virtues required for working in GIS are good communication skills, the ability to understand and resolve problems, and patience. He said that everyone does object oriented design (the problems are broken down into blocks). The Geographical Information Systems magazine and the Association of American Geography magazine are good places to look for want ads.

The mathematical areas of graph theory and topology are the foundation for working in GIS. Mr. Honeycutt said that there are two ways to represent maps. The first way is as a graph, a.k.a. the vector method. The second way is as a grid, a.k.a. the raster method. Here are examples of each:





Mr. Honeycutt specializes in vector techniques. He did a Power Point presentation where he quickly showed the kinds of maps ERSI makes. A few maps were similar to what you would see at Mapquest.

Dr. Derry Connolly (UCSD, until recently at Eastman Kodak)

Dr. Connolly, currently a professor at the University of California in San Diego, spoke with a delightful Irish accent. Unlike the other speakers, he did not talk much about what skills a math major should have, but instead described the Finite Element Method (FEM).

He claimed that the FEM is the most powerful and useful tool in industry, and that we should teach it. In 1982, it was a new method in non-defense industries, but now it is very common.

There are two worlds. In the *math world*, we care about theorems. In *industry*, they care about money. When products like the airplane, automobile, bicycle, and computer were invented, they did not use sophisticated mathematical design. However, anything that is designed today does use it. What has changed since these inventions are

- Powerful tools like FEM.
- Powerful computers.
- Same old math, but new algorithms to exploit the computer.

Many of the mathematical and physics equations were established centuries ago, but could not be solved except for the simple cases. Clough in 1950 epitomized the engineering philosophy, "Let's approximate!" Perhaps math teachers are over concerned with finding exact answers.

FEM uses the fundamentals from mathematics: Linear Algebra, Differentials, and Integral Calculus. There are four steps required for FEM:

Step 1: Identify the differential equation, domain, and boundary conditions. Structures require a balance of forces, heat transfer requires energy transfer, and fluid flow uses conservation of mass. Other examples are electrical current flow, magnetics, and dye diffusion.

Step 2: The differential equation is equivalent to an integral equation. This mathematics is about 2 centuries old.

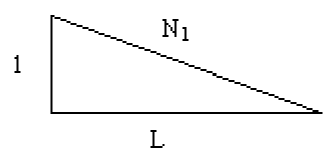
$$\text{D.E.}=0 \text{ in domain } \Omega \quad \Leftrightarrow \quad \int_{\Omega} (\qquad) \, \mu \, d\Omega = 0$$

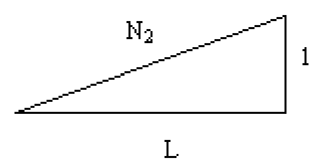
Step 3: Convert the integral equation to a matrix equation. (The matrix may be infinite dimensional.) This mathematics is still quite old.

$$Ku = F$$

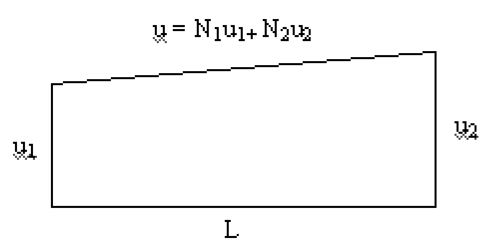
The matrix K will be banded and represents the integral of basis functions; the u is a vector of coefficients of basis functions, like Fourier series coefficients.

Step 4: In the 1950's, the power of this method was unlocked by approximating. (Recall the quote by Clough.) Introduce simple shape functions N_i. The unknown function that we are solving for is $u = \Sigma\, u_i N_i$, and the weighting function is N_i. The shape functions are simple polynomials, and for most physical problems, the shape functions are line segments. For example, you can start with the basic linear polynomials N_1 and N_2:

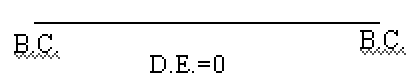


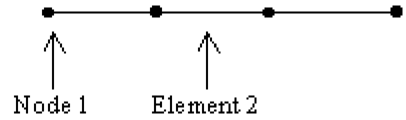


The unknown function will be a linear combination of the two shape functions:



Consider a one-dimensional problem on a line segment:

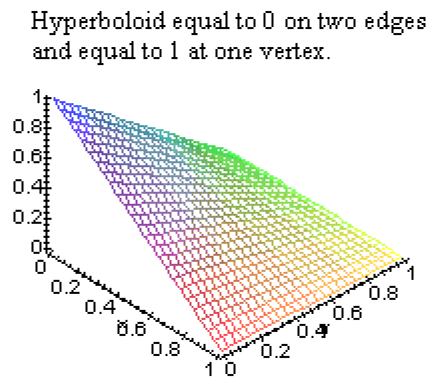


Descretize the domain into 3 *elements* and 4 *nodes*: 

Determining the *element stiffness* k_i for each element is important in FEM. This depends on the property of the material and the shape functions.

A couple of software packages for doing FEM are [ANSYS] and [ABAQUS]. ANSYS costs $95 for students, is free for educators, and is a good package for teaching. Many amateur analysts ignore the mathematics and produce an erroneous pretty picture. This is why it is important for mathematicians to do FEM.

Dr. Connolly gave us a couple of problems to work on in groups. The first problem was to find the simple shape functions over a 2-dimensional square element. The answer turned out to be functions like z = (1-x)(1-y):



The second problem was to figure out what ingredients go into the FEM if you are interested in the flexing of a diving board. One of the ingredients was the cubic shape function because the underlying differential equation is a 4^{th} order beam equation.

Dr. Jeffrey Sachs (Wagner Associates)

Wagner Associates is a company that develops sophisticated software. Dr. Sachs noticed as he passed through airport security that an airport employee wiped his bag with a white disk and inserted the disk into machine that can detect trace amounts of explosives or drugs. That machine uses ion trap mass spectrometry, a topic he discussed later in his talk. He called it, "Biotechnology from bifurcation in a box".

What makes good training for an industrial mathematician:

- Math – solid quantitative reasoning and logical precision, statistics and probability
- Software
- Computer science – fundamental algorithms and analysis principles

He said that mathematicians in industry must avoid the "paralysis of analysis". Another way of putting it is that people who define things rigorously have "rigor mortus".

Necessary communication skills:

- Written/Reading (and research)
- Oral/Aural
- Large groups/Medium groups/One on One
- Formal/Informal talks
- Interpersonal/Intrapersonal

Dr. Sachs said that you must be perceived correctly, and you should speak the language of the client. This is similar to what Dr. Bloom said.

Correct attitude or questions you should ask yourself:

- What is important to my employer?
- What could I enjoy working on?
- What is the best thing I can do here?
- How do I keep my skills fresh?
- How do I keep life in balance?

Those are the questions you should ask yourself no matter what your job is!

The three hottest fields are software, mathematical finance, and biotechnology.

Dr. Sachs recommended putting a Java applet on your web page if you are looking for a job.

Dr. Lisa Bernstein (Genentech)

Genentech is a company that uses recombinant DNA technology to create drugs. They have trained microbes to produce growth hormones, activase, pulmozyme, and anti cancer drugs. Recently, James Dobson, the chairman of Focus on the Family, was administered activase to reverse the deleterious effects of a mild stroke that he suffered. Pulmozyme is a drug for treating cystic fibrosis.

Skills Dr. Berenstein has used:

- Math – numerical methods, statistics, calculus
- Programming – Fortran and matlab
- Biochemistry and physics
- Evaluating software – html conversion for example
- Programming – C, Java Script, cgi
- Graphics

Dr. Berenstein made the following recommendations for changing the math major curriculum:

- Require ordinary differential equations, partial differential equations, numerical methods and analysis, probability and statistics, and **applied** linear algebra.
- More science
- Programming
- Fourier analysis and the idea of spectrum
- Stochastic processes
- Chaos, wavelets, Markov processes
- Applications – imaging, genomics, coding, mathematical population biology
- Internships, jobs, volunteer opportunities, senior project on campus.

Good places to look for the last item are national labs, medical centers, and the Center for the Reproduction of Endangered Species (CRES).

Dr. Bernstein mentioned nuclear magnetic resonance, and finished her talk with several examples.

For the first example, the workshop participants divided up into small groups and solved problems about random walks. She suggested something that I'll try in class: Have a student do the random walk. I can tell the student to move left or right by generating a random number. I did much better on this topic than I did on FEM! We restricted our attention to a fair random walk on the discrete lattice …, -2, -1, 0, 1, 2, … starting at the origin. We found

1. The probability of r rightward steps by time t is $b(r,t) = \binom{t}{r}\left(\frac{1}{2}\right)^t$. (This is just the binomial probability density function with t trials and probability of success equal to ½.)
2. The relationship between the net displacement x(t) and r(t) is x(t)=2r(t)-t. (To get this answer, eliminate the number of left steps L from the system {r – L = x , r + L = t}.)

   We didn't have time to finish the rest of the questions about random walks, but the questions were:

3. Compute $\bar{x}(t)$ and $x_{rms}(t)$ using either Monte Carlo simulations or exact calculations.
4. Compare your answer with the diffusion equation model, $x_{rms}(t) = \sqrt{2Dt}$.
5. The last question involved Einstein's diffusion equation from his famous 1905 paper (A. Einstein, "Investigations on the Theory of the Brownian Movement", Ann. der Physik, v. 17, p. 549, 1905.) on Brownian motion:

$$D = \frac{RT}{6\pi N_A k \rho}$$

where

D = the diffusion constant

R = the gas constant
T = the Kelvin temperature
N_A= Avogadro's number
k = the viscosity of the liquid
ρ = the radius of the Brownian motion particles.

She supplied numbers for the parameters and we were supposed to estimate $N_{A.}$

Answer to 3. One of the workshop participants and I convinced ourselves that it would take too many columns and rows in a spreadsheet to do a Monte Carlo simulation. Here is the exact computation:

$$\bar{x}(t) = 2 < r(t) > -t = 2\left(t\frac{1}{2}\right) - t = 0$$

This says that the expected location of a fair random walk starting at 0 is 0. RMS means "root mean square". In the case where the mean is zero, $x_{rms}(t)$ is the same thing as the standard deviation. Here's how to simplify $x_{rms}(t)$:

$$\begin{aligned} x_{rms}(t) &= \sqrt{< x >^2} \\ &= \sqrt{Var(x)} \\ &= \sqrt{4Var(r)} \\ &= \sqrt{4(t \cdot \tfrac{1}{2} \cdot \tfrac{1}{2})} \\ &= \sqrt{t} \end{aligned}$$

Answer to 4. Comparing this result to the diffusion equation model, we see that the diffusion constant D is ½.

I was not interested in solving problem 5.

For the second example, we were to use the Monte Carlo method to find a definite integral. (I tell my probability students about using Monte Carlo for finding areas, but I think I will make it an assignment this fall so they can see this wonderful application of randomness. Dr. Bernstein explained the Monte Carlo method as follows. Suppose u is a continuous function on the interval [a,b]. Randomly select N points on [a,b] using the uniform distribution. The "fundamental theorem" of Monte Carlo is

$$\int_a^b u\,dx \approx (b-a)\left[< u > \pm \frac{\sigma_u}{\sqrt{N}}\right]$$

where <u> is the mean of the u values of the randomly selected x values. We walked to a computer lab and used Microsoft Excel to estimate the erf(1) where erf is the error function:

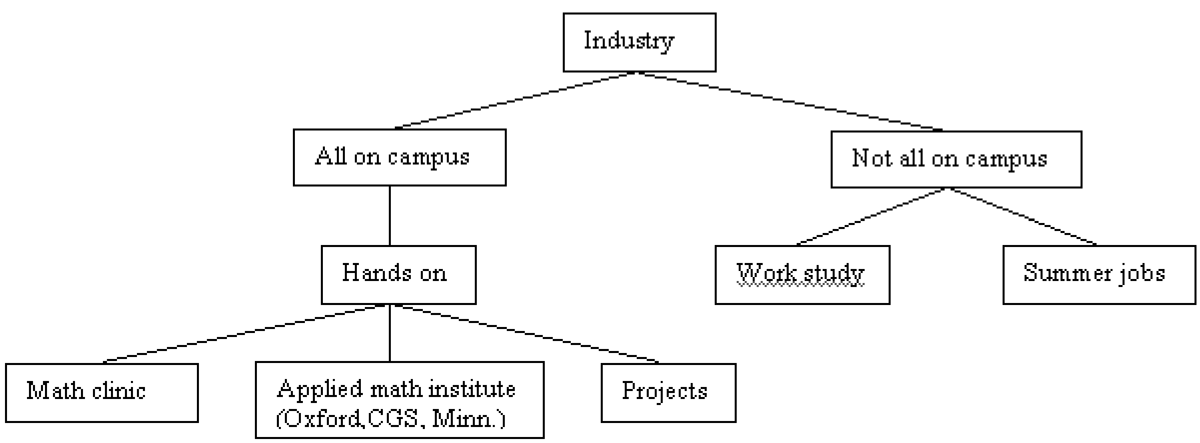
$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-t^2\right)\, dt$$

I put 500 random numbers chosen from [0,1] into column A, the integrand evaluated at these numbers in column B, and the square of column B in column C. The estimate for erf(1) is the average of column A and I obtained .830. I also obtained .0104 for the error in estimating erf(1). Dr. Bernstein asked what the easiest method for finding erf(1) would be, saying that this would be the method used in industry. I used the built-in erf function in Excel and got .843! See the Microsoft Excel 97 spreadsheet for the complete example.

The Panel Discussion

The panelists consisted of two earlier speakers, Dr. Lisa Bernstein and Dr. Raymond Roan , and Dr. Robert Borrelli (Harvey Mudd College), Dr. Gary Green (Aerospace Corporation), and Dr. Lance Small (National Science Foundation and UCSD). At this stage of the conference, there was an emphasis on how to get colleges and industry to cooperate.

The National Science Foundation offers grant opportunities for collaboration with industries. Dr. Borelli, whose math department is heavily involved in this, outlined the ways that a university can be involved with industry:



The math clinic is the type of activity that Dr. Borrelli is involved in. He finds a sponsor from industry, business, or government. Three to five students work on a project that lasts for an entire year and has value for both the students and the sponsor. The students give oral and written reports periodically, and travel to the company at least once. The sponsor pays a fee of $36K per project. That seems like a lot, but he said that it does not make the department any money. The Colorado School of Mines is one sponsor that Dr. Borrelli has found. He and the other participants said **we** should take the initiative in making industry contacts.

Dr. Green commented about what was important at the Aerospace Corporation:

- Be a lifetime learner.
- Possess communication skills.
- Have cross-disciplinary skills and interests.
- Have problem solving abilities (timeliness, partial solutions, posing/defining problems).

Dr. Small recommended offering minors or variations in math degrees, for example, biomathematics. He also suggested that a student read their proofs aloud to a friend, and if they do not have any friends, they can speak into a mirror. ☺

One of the workshop participants, Glen Ledder from the University of Nebraska-Lincolon, has done research on mathematics in industry and would be a useful resource.

Biography

Michael Lloyd received his B.S in Chemical Engineering in 1984 and his Ph.D. in Mathematics from Kansas State University in Manhattan. He has presented papers at meetings of the American Mathematical Society and the Mathematical Association of America. He has been at Henderson State University since August 1993.